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Avalanche statistics of sand heaps

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Abstract: Large scale computer simulations are presented to investigate the avalanche statistics of sand piles using molecular dynamics. We could show that different methods of measurement lead to contradicting conclusions, presumably due to avalanches not reaching the end of the experimental table.

Keywords:

The physics of an evolving sandpile has been of large interest to physicists and engineers and there has been done much work in this field. One of the most popular (or sometimes unpopular) ideas is the concept of self organized criticality (SOC) [1]. It has been argued by many physicists that sandpiles can be described by cellular automata in two or three dimensions (e.g. [2]) and by stochastic cellular automata (e.g. [3]) which in simulations might show SOC-behavior. There are many effects in nature which are supposed to reveal SOC, and hence there is a variety of articles investigating the theory of SOC (e.g. [4]). When particles are dropped one after the other onto the top of a sand heap one observes avalanches. The time intervals in between successive avalanches and the size distribution of the avalanches have been of large interests to many scientists. Experimentalists as well as theorists investigated these quantities, and there is a controversy whether they obey a power law or not [5,6].

In an experimental work Jaeger et al. [7] investigated the avalanche sizes of a pile contained in a box with one open side. The material flow over the edge of the box was measured in between a pair of capacitor plates. From the fluctuation of the capacity they concluded that the sizes of the avalanches

might *not* be power law distributed. They count for the size of an avalanche the capacity change, i.e. the mass of the particles which fall down over the edge of the table. Bretz et al. measured the avalanche distribution by recording the temporal behavior of the inclination of the heap's surface [8]. Contradicting to [7] their results support the hypothesis of the power law distribution.

We want to present the results of a large scale computer experiment where we recorded the distribution of the sizes of avalanches using both methods. Finally we will show that, perhaps, both measurements [7,8] do not contradict, but support each other.

In a previous paper [9] we have shown that in simulation using two dimensional molecular dynamics of non spherical particles one can find a power law behavior of the size distribution of the avalanches. Our particles k have been built up of five spheres which are connected by springs (fig. 1).

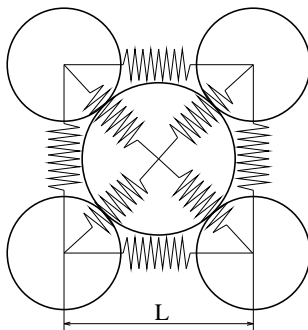


Fig. 1. Each of the non spherical particles consists of five spheres.

At rest the inner sphere touches the surrounding spheres of the same particles. For details of the forces acting between the particles of the same grain via springs and the forces acting between colliding grains see [10].

Using molecular dynamics we build up the pile by dropping the particles one after the other on the top of the evolving pile. A particle was dropped when all avalanches and fluctuations caused by the previously released grain have faded away, i.e. we waited until the maximum velocity of the particles comes very close to zero. Then the inclination of the heap was measured due to the following procedure: Suppose the shape of the heap of height H built up on a surface of width B is close to a triangle. Then its slope is given by

$$\Phi = \arctan \left[\frac{H - \frac{1}{M} \sum_{i=1}^N m^{(k)} y^{(k)}}{\frac{2}{M} \sum_{i=1}^N m^{(k)} x^{(k)}} \right], \quad (1)$$

where $m^{(k)}$, $x^{(k)}$ and $y^{(k)}$ the mass and the position of the k th grain and M is the sum of the masses of all particles $M = \sum_{i=1}^N m^{(k)}$. Since our heap is close

to, but not an ideal triangle we calculated the height H using

$$H = \frac{2}{x_{max}} \int_0^{x_{max}} h(x) dx, \quad (2)$$

where x_{max} is the x -position of the grain which is closest to the end of the table. From the fluctuations of the slope due to eq. (1) we can conclude the approximate size of the avalanche according to a decrease in the slope:

$$\Delta^{(1)}M = \frac{B^2}{2} \cdot \rho(\tan \Phi' - \tan \Phi), \quad (3)$$

where Φ' is the slope before the dropping event, $B \approx P = 30.7 \text{ cm}$ is the length of the table and $\rho = 0.59 \text{ g} \cdot \text{cm}^{-2}$ denotes the average density of the heap. This method for the measurement of the size of an avalanche (indexed by $\Delta^{(1)}M$) is close to the experimental method used by Bretz et al. [8]. Another method which was used by Jaeger et al. [7] and by Rosendahl et al. [6] is to measure the weight of the material which reaches the end of the finite table during an avalanche using a balance or a capacitor. They considered the weight of material flowing over the border of the table to be the size of the avalanche $\Delta^{(2)}M$. Similar as in the experiment in the molecular dynamics simulation we calculated the mass of the particles which reach the end of the table, i.e. $x^{(k)} > P$.

Collecting the idle time of all computers of our department over a period of about one year we found enough computer power to perform a large scale molecular dynamics simulation. The heap was build up on a rough surface of width $P = 30.7 \text{ cm} \approx 200 \cdot \langle r_i^{(k)} \rangle$. The radii $r_i^{(k)}$ of the outer spheres of the particles k were equally distributed in the interval $r_i^{(k)} \in (0.05, 0.11) \text{ cm}$ while the radii of the inner spheres are determined by the relation $r_m^{(k)} = L^{(k)} / \sqrt{2} - r_i^{(k)}$ ($i = 1 \dots 4$). $L^{(k)}$ is the size of the k th grain (s. fig. 1). In [10] (fig. 10) we have shown that the relation $\frac{L^{(k)}}{r_i^{(k)}} = 4$ reproduces well the static friction behavior of sandpiles. The average number of particles on the heap was $N_{av} = 930$.

Figure 2 shows the time series of the avalanche size due to both procedures, $\Delta^{(1)}M$ and $\Delta^{(2)}M$. The size distributions of the data shown in fig. 2 are drawn in fig. 3. We find that the distribution of the avalanche sizes measured by means of the fluctuations in the slope $\Delta^{(1)}M$ reveals a typical power law behavior for avalanches smaller than $\Delta M^{(1)} < 2$ grams, while the distribution according to the direct measurement of the avalanche sizes $\Delta^{(2)}M$ does *not* show a power law behavior. We claim that the difference in both figures (3) comes from the fact that the second method is not able to care for those avalanches which do not reach the end of the table. Obviously the larger the pile the higher is the fraction of avalanches which do not reach the border of the table. This

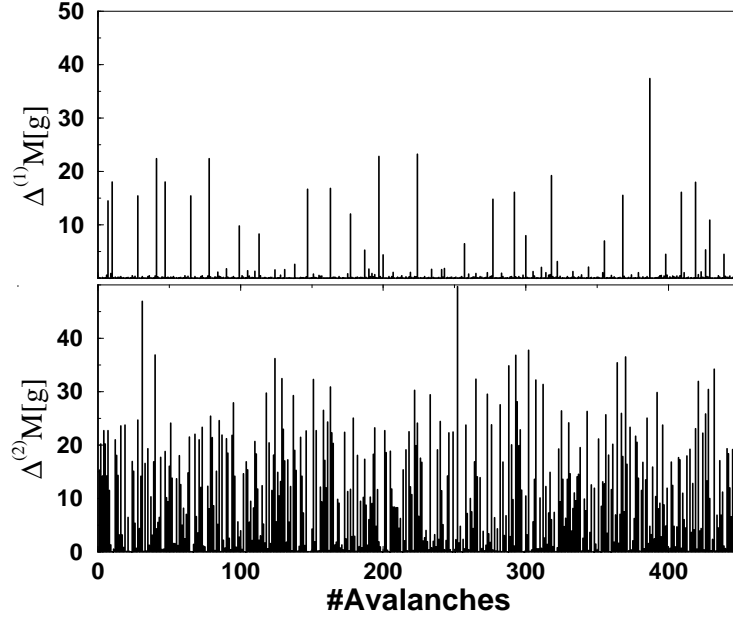


Fig. 2. Series of 450 avalanches. Top: the avalanche size $\Delta^{(1)}M$ concluded from the fluctuation in slope (equation (3)), bottom: the avalanche size $\Delta^{(2)}M$ calculated from the mass of particles that reach the end of the table. The fraction of small avalanches is much higher for the upper figure.

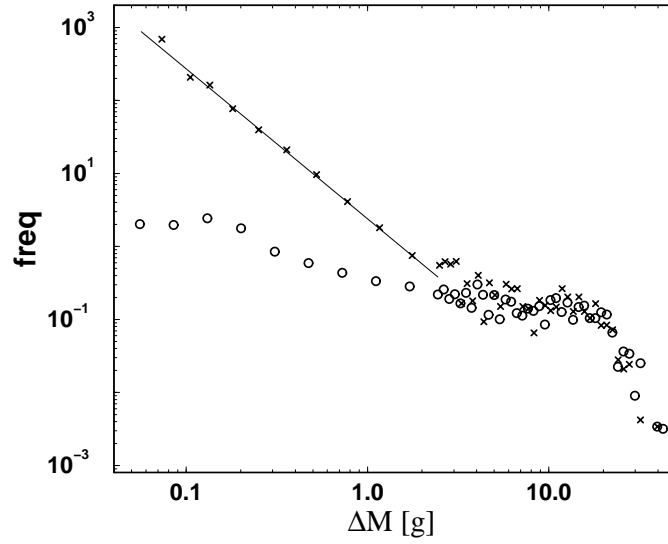


Fig. 3. The size distribution of the avalanches in log-log-scale due to $\Delta^{(1)}M$ (\diamond) and $\Delta^{(2)}M$ (+), respectively. The sizes are measured in grams. The line shows the function $freq \sim \Delta^{(1)}M^{-1.85}$

coincides with the observations by Jaeger et al. [7] who found a deviation from the power law scaling for *large* systems. They found a sharply peaked avalanche distribution of large, system overspanning avalanches.

For large avalanches in our simulation both types of measurements lead to very similar results which supports our conclusion. In agreement with the experimental observations by Rosendahl et al. [6] we find large avalanche tails in the distribution. In the case of large piles, the direct measurement of the mass fluctuations, i.e. neglecting the smaller avalanches, would lead to similar results as Jaeger et. al. found.

For the waiting time distribution of the large avalanches, i.e. for the distribution of the number of dropping events in between two consecutive large avalanches, surprisingly we find a double peak. Fig. 4 shows the distribution for five different sizes of the discretization intervals. The double peak structure is found in all five curves, hence we assume that it is not an artifact due to the choice of the size of the discretization interval. So far we have no explanation for this behavior.

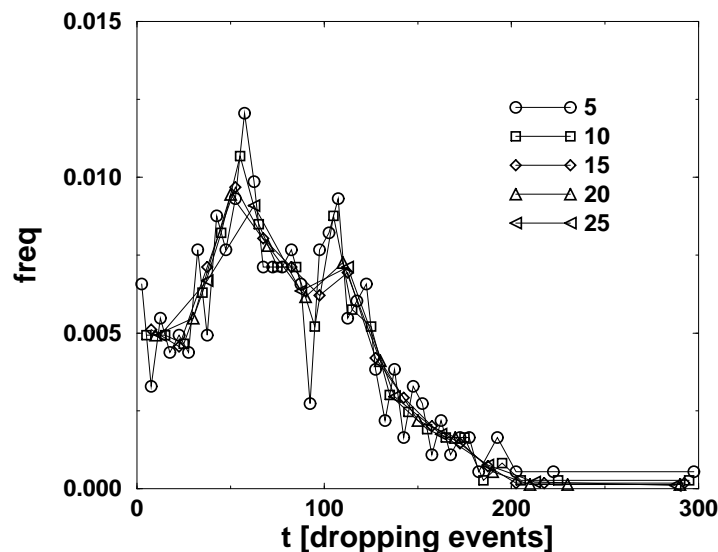


Fig. 4. The waiting time distribution of the intervals in between two consecutive large avalanches. The figure shows the number of pairs of avalanches over their time distance measured in dropping events. The double peak structure remains preserved for different discretization interval sizes.

Although our simulation seems at least not to contradict the concept of SOC we should remark here that there are other serious objections against applying the idea of SOC in the case of sand pile avalanches (see e.g. [11]).

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